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CHARACTERISTICS OF WIND-INDUCED WAVES IN A SHALLOW WATER ZONE

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ABSTRACT
The parameters determine waves energy in shallow water zone that pronounces the crucial influence on abrasion of both natural and artificially paved banks. The effort to re-development of the relations was found as absolutely necessary for waves energy calculations. Substantial benefit of the work is found not only in enabling the use of computers while avoiding time-consuming and difficult application of diagrams, but namely in recent recognition that the calculation results showed a risk of underestimate the real impact of wind-induced waves. In some cases, the calculations respecting the above standard produce lower values of waves height and time-period and thus also lower values of wave energy.

KEYWORDS: height, length and time-period of the waves, energy of waves, shallow water zone

1. INTRODUCTION

There is a Czech standard ČSN 750255 valid for calculation of wind-induced water waves, however, in times when the standard was introduced; modern computers had not been widely available for project designers. This is why auxiliary diagrams had been used for calculations needed for solving of rather difficult calculations relevant to the problem. The above standard had introduced diagrams used for calculation of waves time period and height originating from former Soviet Union as seen on Figs. 3 and 4. Just very recently it was recognized that, most probably, both diagrams were overtaken from literature source (Jahoda, 1997) without referring to relevant formula, giving results in real coordinates for wind velocities 20 and 30 $m.s^{-1}$. Also, it has not been possible to find what procedure was originally used for passing from the diagrams to the dimensionless coordinates of wave height, time-period and length.

Actually, unknown relations on whom the above-mentioned diagrams were developed hamper the use of computers. This concerns namely of wave parameters in shallow water. The parameters determine waves energy in shallow water zone that pronounces the crucial influence on abrasion (Lukáč and Abaffy, 1980; Kratochvil, 1969) of both natural and artificially paved banks. Whatever complicated and/or time-consuming, the effort to re-development of the relations was found as absolutely necessary for waves energy calculations.

Substantial benefit of the work is found not only in enabling the use of computers while avoiding time-consuming and difficult application of diagrams, but namely in recent recognition that the calculation results showed a risk of underestimate the real impact of wind-induced waves. In some cases, the calculations respecting the above standard produce lower values of waves height and time-period and thus also lower values of wave energy. The presented study is therefore focussed on calculations of main wave parameters in shallow water zones.

2. WAVE PARAMETERS IN DIMENSIONLESS NUMBERS

Height, length and time-period of the waves occurring on water surface is dependent on reservoir size, wind velocity, water depth and reservoir bottom shape (Spanilá et al., 1999). In order to avoid the influence of bottom shape that is impossible to describe by numbers, all measurements had been performed on deep water. According to the above standard, this concerns any depth bigger than one-half of the wave length. Due to general expensiveness of the measurements, methods were searched for, how to generalize and use the measurement results not only local but also foreign authors. Thus, for example, influence of the reservoir sizes and wind velocity is described by a dimensionless number $\boldsymbol{s}$ and the wave height by a number $\boldsymbol{\nu}$. The following relations define these dimensionless characteristics:

$$v = \frac{gh}{w^2} \quad , \quad t = \frac{gT}{2\pi w} \quad , \quad b = \frac{gH}{w^2} \quad , \quad s = \frac{gD}{w^2} \tag{1}$$

The measurement of wave length is not performed as far as it was shown that the measurement itself is not only extremely difficult but also much less reliable than that of the wave time-period and height. The determination of the wavelength mean value based on calculations is more reliable.

According to the above Czech standard, the calculation procedure is as follows: For given water depth H, the length of wind route D and its velocity w, characteristic numbers s and b are calculated for which relevant dimensionless characteristics $\nu = gh/w^2$ and $t = gT/2\pi w$ are found on diagrams in Figs. 3 and 4, leading further on to calculation of wave height h and time-period T. The wavelength calculation using progressive-approach method is described in the standard.

At the initial phase - searching for relations, according to whom curves were constructed in the mentioned diagrams, the only requirement was considered, e.g. technically sufficient fitting of developed formulas with curves in Figs. 3 and 4. Finally, the following relations were found:

$$t \approx t_s \; tgh \frac{\sqrt{b}}{t_s} \tag{2}$$

For the wave height

$$v \approx v_s \; tgh \frac{\delta \sqrt{b}}{v_s} \quad , \quad \delta = 0.221 b^{0{,}312} \tag{3}$$

Symbol ν_s is used for the dimensionless height, while symbol t_s for dimensionless wave time-period in deep water, theoretically for $H \to \infty$. Relevant curves in diagrams in Figs. 3 and 4 can with sufficient accuracy be described as the following functions of parameter s:

$$\log \; s = -2.4522 + 0.4478. (\log s) + 0.00926 (\log s) - 0.007 (\log s)^3 \tag{4}$$

$$\log ts = -1.05 + 0.2656. (\log s) + 0.00926 (\log s) - 0.005 (\log s)^3$$

Logarithms used in the above equations are decade ones.) Fitting of the relations (2) and (3) vs. curves in Figs. 3 and 4 is relatively good, however, the accordance has not been reached in the whole extent of the curves. This concerns namely partial curve shape for b = $gH / w^2 \leq 0.04$, that is for $s \geq 1000$ unlikely flat.

3. WAVE PARAMETERS CALCULATIONS IN SHALLOW ZONE

The following general relation (ČSN, 1985) is valid for the wave motion on water surface of the final depth H:

$$\lambda = \frac{gT^2}{2\pi} \, tgh \frac{2\pi H}{\lambda} \tag{5}$$

Theoretically, the former relation is valid for reservoirs with a flat bottom, without an excessive error it can be applied for mild sloped banks, abrasion of whose is mostly analysed. For wind-induced waves on deep water, theoretically for $H \to \infty$, practically for depth interval of about 7 to 10 m, when the bottom shape influence is disappearing, the following relation is valid between time-period T_s and wavelength λ_s:

$$\lambda_s = \frac{gT_s^2}{2\pi} \tag{6}$$

The measurement results are expressed as dimensionless numbers (Spanilá et.al., 1999). While still missing in relation (1), for the dimensionless characteristic of the wave length the following relation was derived:

$$l = \frac{g\lambda}{w^2}$$

Now, general equation (5) can be transformed into dimensionless form, e.g.

$$l = 2\pi \, t^2 \; tgh \frac{2\pi \, b}{l} \tag{7}$$

The equation is described in Fig. 1 for deep water with curve b, then for only a little smaller depth with curve b - Δb, and for shallow water with curve b = 0.04. A procedure is indicated in upper right corner of the Fig. 1, how to shift from the depth b to that of b - Δb which is explained in the next indent:

Relation (7) is to be modified as follows:

$$t = \sqrt{\frac{l}{2\pi \; tgh \dfrac{2\pi \, b}{l}}} \tag{8}$$

The following operations are focussed on finding what change undergoes the time-period t and length l when a small depth change from b to b - Δb occurs. As far as the formula can be solved with numerical methods only, the differentials are to be

$$\begin{aligned} dt &\approx \Delta t \approx (t_1 - t) \\ dl &\approx \Delta l \approx (l_1 - l) \\ db &\approx \Delta b \approx (b_1 - b) \end{aligned} \tag{9}$$

The indices 1 (one) belong to the depth $b_1=b-\Delta b$. The procedure then continues so that out of known parameters t and l for deep water *(b)* parameters t_1 and l_1 are step-by-step calculated for depths only a little bigger *(b_1)* - i.e. each depth change Δb possess a negative sign. In our case the calculations are to be terminated when depth $b = 0.01$.

When the depth is diminished by Δb, the period is changed by Δt and the length by Δl. Interrelations between the changes is obtained when differencing of equation (8), thus

$$\Delta t = F\Delta l + K\Delta b \tag{10}$$

where

$$F = \frac{\partial t}{\partial l};\; K = \frac{\partial t}{\partial b} \tag{11}$$

The new parameters t_1, l_1 and b_1 have to lie both on line (10), and on line $b - \Delta b = b_1$, the further describing relations between the new parameters - see point X on the right upper corner of Fig. 1. At the same time, besides equation (10) also the following equation has to be satisfied

$$t_1 = \sqrt{\frac{l_1}{2\pi\, tgh \dfrac{2\pi\, b_1}{l_1}}} \tag{12}$$

Referring to relations (9), finally it is derived that for already known depth b_1 the searched parameters t_1, l_1 are obtained by solving of the following equation

$$\sqrt{\frac{l_1}{2\pi\, tgh \dfrac{2\pi\, b_1}{l_1}}} - F l_1 = \sqrt{\frac{l}{2\pi\, tgh \dfrac{2\pi\, b}{l}}} - F l - K\Delta b \tag{13}$$

The solution primarily leads to length l_1, while the time-period t_1 is determined by the expression found under the radical on the left side of the equation - see also relation (12).

The expressions F and K are slightly more complicated - they are partial derivations of a composed function. Their initial relation is to be written as follows:

$$t = h(u,v) \tag{14}$$

where

$$h = \sqrt{\frac{u}{v}} \quad ; \quad u = \frac{l}{2\pi} \quad ; \quad v = tgh(z) \quad ; \quad z = \frac{2\pi\, b}{l}$$

The following relations are valid for expressions F and K:

$$F = \frac{\partial h}{\partial u}\frac{\partial u}{\partial l} + \frac{\partial h}{\partial v}\frac{\partial v}{\partial l} \tag{15}$$

where

$$\frac{\partial h}{\partial u} = \frac{1}{2}\sqrt{\frac{z}{b\, tgh(z)}} \quad ; \quad \frac{\partial u}{\partial l} = \frac{1}{2\pi}$$

$$\frac{\partial h}{\partial v} = -\frac{1}{2}\sqrt{\frac{b}{z}}.\frac{1}{tgh(z)\sqrt{tgh(z)}} \quad ; \quad \frac{\partial v}{\partial l} = -\frac{z^2}{2\pi b \cosh^2 z}$$

$$K = \frac{\partial h}{\partial u}\frac{\partial u}{\partial b} + \frac{\partial h}{\partial v}\frac{\partial v}{\partial b} \tag{16}$$

While $\partial u / \partial b = 0$ and $\partial h / \partial v$ is already known from the precedent equation, the following needed to be determined in addition:

$$\frac{\partial v}{\partial b} = \frac{z}{b \cosh^2(z)}$$

It remains now to determine, what value b might be considered as deep water, and how big iteration step is to be chosen in order to reach the most possible accuracy of the calculation. According to the mentioned standard, deep water includes each depth greater than one half of the wavelength. More stringent criterion was chosen in our study - any depth that is bigger than twice wavelength characterizes deep water (Spanilá et al., 1999) Selection of suitable iteration step was determined empirically - see Fig. 2. The largest differences in the time-period t calculations can be found in the case when $s = 200$, even in rather different iterations. In the first case for $\Delta b/b = 0.05$, in the second one for $\Delta b/b = 0.5$. Detailed analysis showed that optimal selection lies between 10 and 20 percent - here it is considered 15 percent leading to $\Delta b / b = 0.15$. Calculation of wave heights is based on the assumption that slenderness ratio of waves expressed as λ/h remains practically unchanged with changes of water depth. This is also valid for ratio of relevant dimensionless characteristics l and v, thus

$$\frac{l}{v} = \frac{2\pi\, t_s^2}{v_s} \tag{17}$$

Dimensionless time-period t_s and height v_s belongs to values measured on reservoir deep water characterized by parameter s. Ratio (17) might be derived from relations (1) and (5) to (7).

In introduction it was stated that in some cases smaller wave heights and time-periods are obtained according to the standard, however, mostly it is true only for one of the two parameters, i.e. either height, or time-period (Krasnožon and Sidorova, 1961). Whatever, this or that, the risk of wrong determination of wave energy is in neither case diminished. It is necessary to decide how to modify or amend the standard according to the actual state of computer technology. Besides substituting of diagrams in

Figs. 3 and 4 for diagrams in Figs. 5 and 6, there are generally two possibilities. The first is to find for *t* and ***v*** relations similar to those given ad (2) and (3) expressing with sufficient accuracy relations given in Figs. 5 and 6. The second is to present required relations in the standard so that any designer is able to prepare relevant computer program. The authors of this paper consider the first possibility as more suitable one.

Determination of equation (13) roots is described in Fig. 7. The roots should be searched for within an interval less then 1/100 000. This explains that only a very approximate solution could be found in the times when the standard was introduced.

4. CONCLUSIONS

Calculations of bank pavement - eventually of their abrasion when not paved - are based on waves energy in littoral area, i.e. mostly in shallow water. Calculations performed according to the standard (ČSN 750255, 1985) need extremely time-consuming search in supplemental diagrams. A number of alternative calculations have to be performed for dimensioning of bank pavements, so for relevant calculations suitable computer program should be developed. This further requires knowing relations on which drawing of supplemental diagrams presented in Figs. 3 and 4 was based. The standard was initially prepared on data from former Soviet Union, and thus, efforts (Krasnožon and Sidorova, 1961) to acquire the original data failed. Nothing else remained than to try to develop the missing relations once more. After several failures there came a success that is shown in this study. Substantial benefit of the work is not only in enabling computer calculations but also in results demonstrating possible risks of underestimating of wind waves impacts. In some cases, the calculations respecting the standard produce lower values of wave height and/or wave time-period and thus, also, lower values of wave energy.

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LIST OF SYMBOLS

D	- wave course length [m]
H	- water depth to bottom [m]
w	- wind velocity [ms^{-1}]
h	- wave height [m]
λ	- wave length [m]
T	- wave time-period [s]
b	- dimensionless depth [-]
v	- dimensionless wave height [-]
t	- dimensionless wave time-period [-]
g	- gravity gradient [ms^{-2}]

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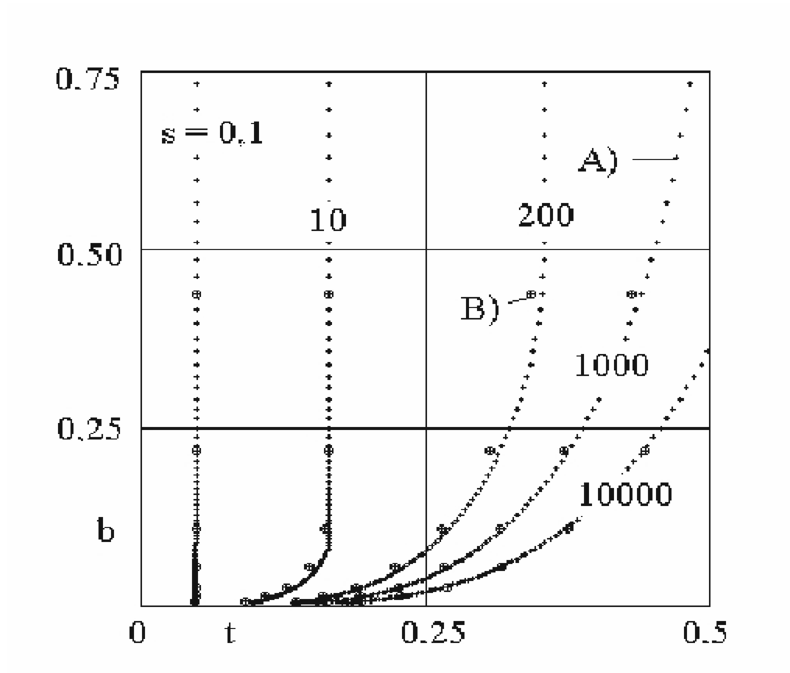

Fig. 1 Calculation of changes in time-period *t* and length *l* vs. small changes of depth from *b* to *b1=b -Δb*.

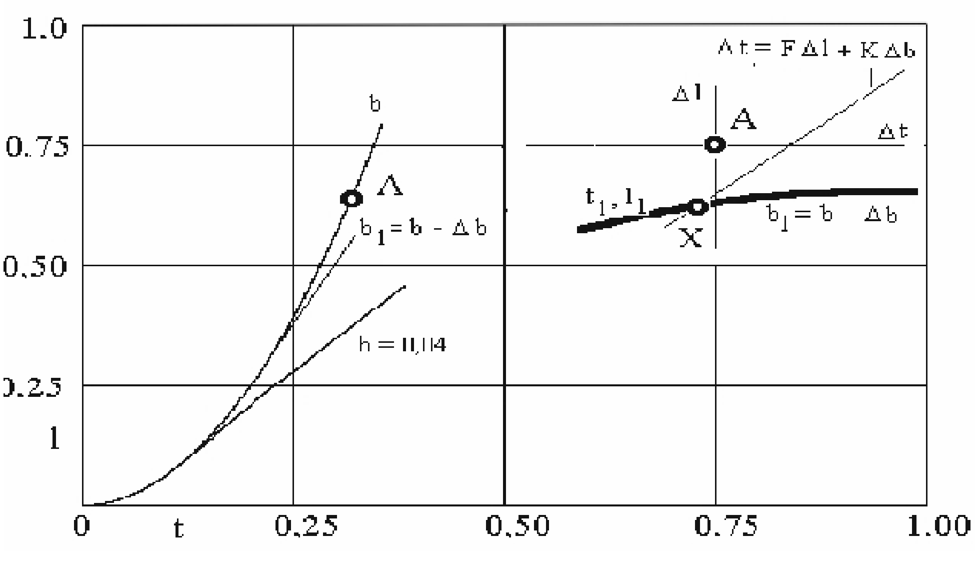

Fig. 2 Accuracy of calculations vs. iteration steps equal to.
A) ***Δb/b*** = 0,05 ***t*** – time-period
B) ***Δb/b*** = 0,5 *b* – depth

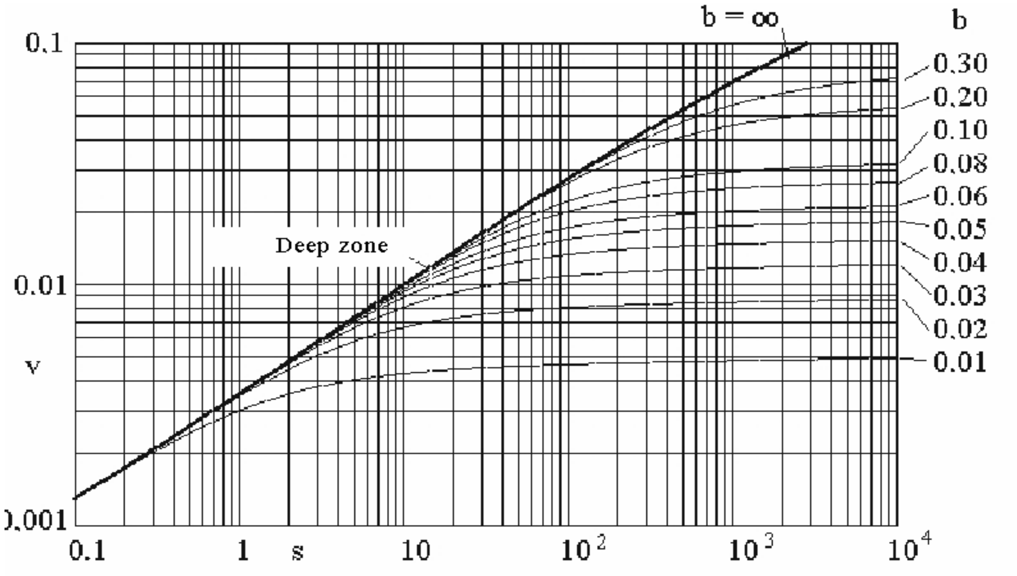

Fig. 3 Graph for determination of characteristic wave height *h* in deep and shallow zone as dependent on parameters $s = gh / w^2$ and $b = gH / w^2$ (according to standard ČSN 75 0255).

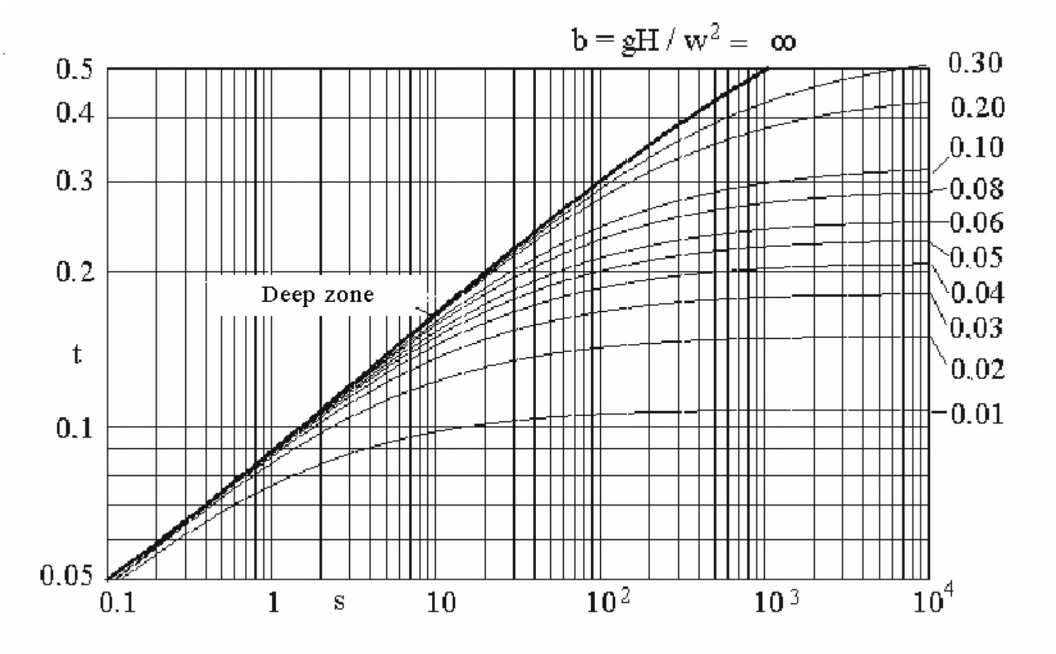

Fig. 4 Graph for determination of characteristic wave time-period t in deep and shallow zone as dependent on parameters $s = gh / w^2$ and $b = gH / w^2$ (according to standard ČSN 75 0255).

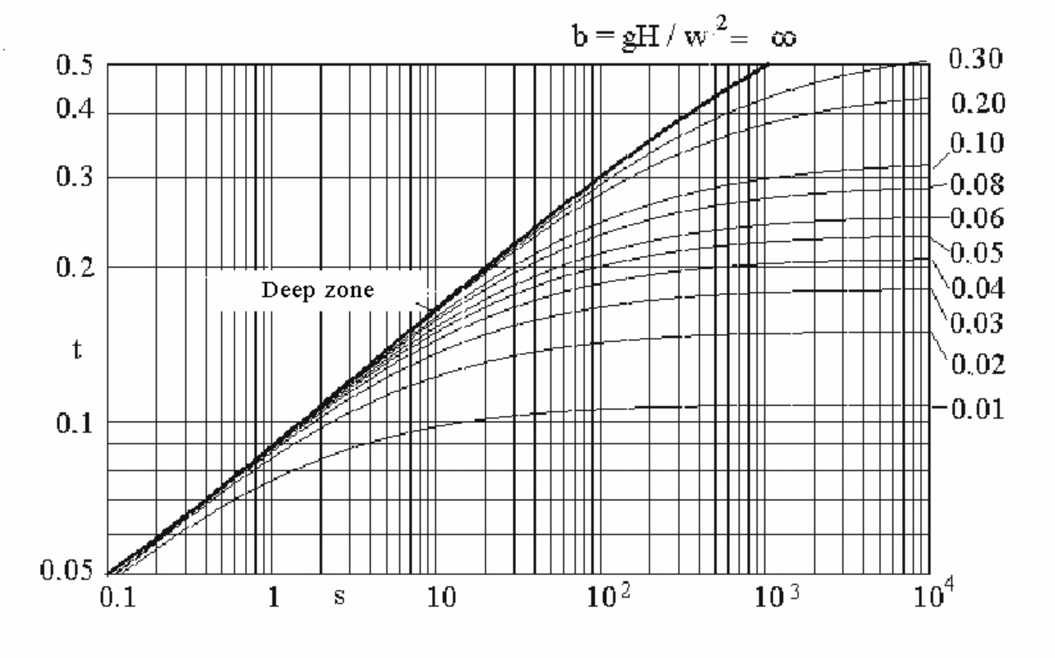

Fig. 5 Graph for determination of characteristic wave height h in deep and shallow zone as dependent on parameters $s = gh / w^2$ and $b = gH / w^2$ (Accurate calculation).

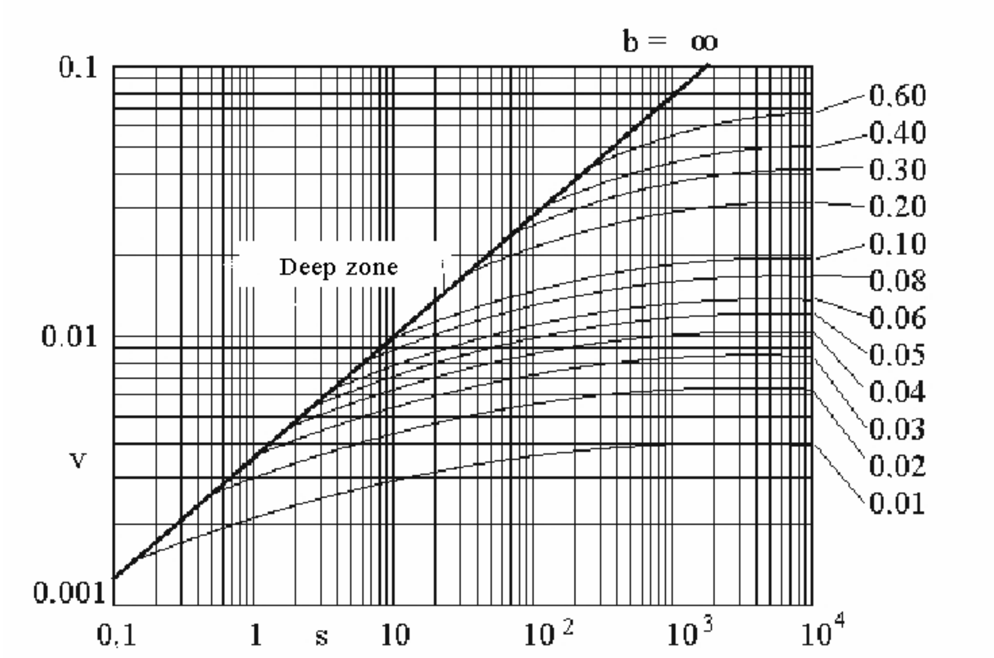

Fig. 6 Graph for determination of characteristic wave time-period t in deep and shallow zone as dependent on parameters $s = gh / w^2$ and $b = gH / w^2$ (Accurate calculation).

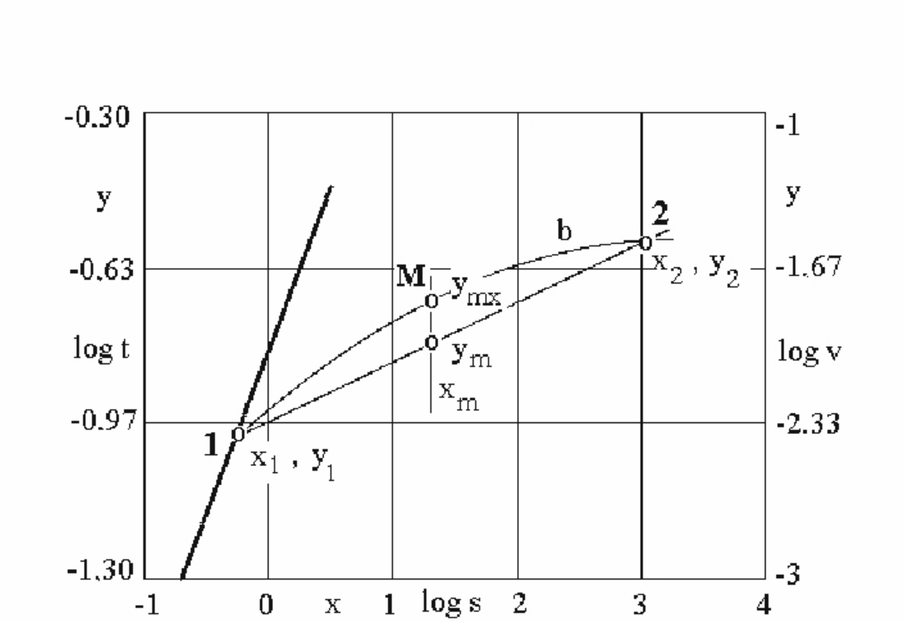

Fig. 7 Scheme to approximation of time-period dependence $t = gT/(2\pi w)$ and height $v = gh/w^2$ on parameter $s = gh / w^2$ for water depth $b = gH / w^2$, when $x = log\ s$, $y = log\ t$.

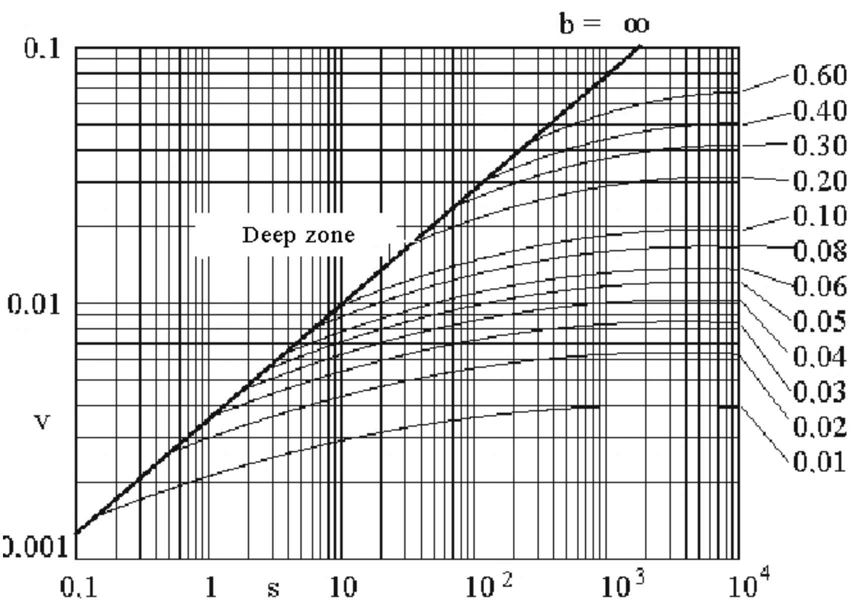

Fig. 8 Approximation of dependences when calculating wave height $v = gh/w^2$, as shown exactly on Fig. 5 $s = gh / w^2$, $b = gH / w^2$.

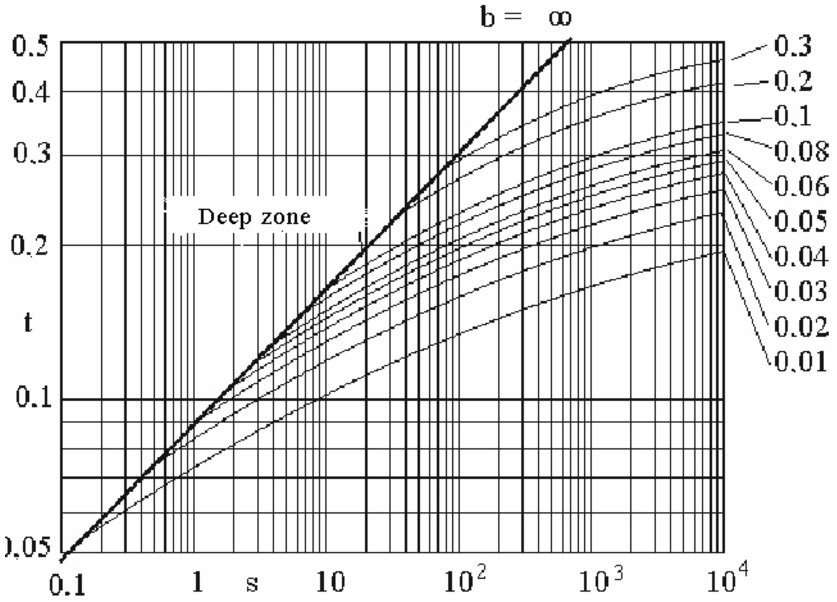

Fig. 9 Approximation of dependences when calculating wave time-period $t = gT/(2\pi w)$, as shown exactly in Fig. 6 $s = gh / w^2$, $b = gH / w^2$.